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On Grids in Topological Graphs

A *topological graph* is a graph drawn in the plane with vertices represented by points and edges as arcs connecting its vertices. If the edges are drawn as straight-line segments, then the graph is *geometric*. A (k, l)-grid in a topological graph is a pair of edge subsets E_1 and E_2, such that $|E_1| = k$, $|E_1| = l$, and every edge in E_1 crosses every edge in E_2. It is known that for fixed constants k, l, every n-vertex topological graph with no (k, l)-grid has $O(n)$ edges. We conjecture that this remains true even when: (1) considering grids *with distinct vertices*; or (2) the edges within each subset of the grid are required to be *pairwise disjoint* and the graph is geometric. These conjectures are shown to be true apart from $\log^* n$ and $\log^2 n$ factors, respectively. We also settle the second conjecture for the first nontrivial case $k = 2$, $l = 1$, and for *convex* geometric graphs. The latter result follows from a stronger statement that generalizes the celebrated Marcus-Tardos Theorem on excluded patterns in 0-1 matrices.